Example Final Exam

Philosophical Logic 2025/2026

Disclaimer

This example exam aims to be representative of the final exam, even though the final exam might be different (different distribution of points, different topics, ...). The exam is closed-book, and a list of relevant definitions can be found after the exercises. Please note that the list provided for the final exam may differ slightly, as it will only include definitions pertinent to the specific exercises on the exam.

To make the most of this practice exam, set aside two uninterrupted hours to simulate the actual exam conditions. This differs significantly from completing homework assignments, where you typically have more time to think about the exercises and write down your answers.

After completing the exam on your own, feel free to discuss it with others.

The first three exercises are formal. The final exercise is more philosophical in nature.

Exercise 1 [30 points]

Show that:

1. **Strong Kleene:**
   $\neg(p \wedge q) \vDash_{K_3^s} \neg p \vee \neg q$ and $\neg p \vee \neg q \vDash_{K_3^s} \neg(p \wedge q)$
2. **Counterfactuals:** $p \rightsquigarrow q \nvDash \neg q \rightsquigarrow \neg p$
3. **Supervaluations** (global consequence relation):
   Show that the following meta-inference (contraposition) fails: if $\varphi \vDash_g \psi$, then $\neg\psi \vDash_g \neg\varphi$.
   (i.e., find formulas φ, ψ s.t. $\varphi \vDash_g \psi$, but $\neg\psi \nvDash_g \neg\varphi$).
4. **Non-monotonic logic:**
   Show that the following rule cannot be derived in **P**: if $\varphi \mathrel{|\!\sim} \psi \supset \chi$, then $\varphi \wedge \psi \mathrel{|\!\sim} \chi$.
   (i.e., provide a preferential model M and formulas φ, ψ, χ s.t. $\varphi \mathrel{|\!\sim}_{\mathcal{M}_P} \psi \supset \chi$, but $\varphi \wedge \psi \mathrel{|\!\not\sim}_{\mathcal{M}_P} \chi$).
5. **Probabilistic entailment:**
   $p \to (q \supset r) \nvDash_P (p \wedge q) \to r$, where '$\to$' is the indicative conditional, defined using conditional probability $P(\varphi \to \psi) = P(\psi|\varphi) = \frac{P(\psi \wedge \varphi)}{P(\varphi)}$.

Exercise 2 (20 points)

Consider the fuzzy logic $Ł_{\aleph_1}$ (truth values $[0, 1]$) with preservation of truth $\vDash_1$ and Łukasiewicz semantics. So $\Gamma \vDash_1 \varphi$ iff for any fuzzy valuation v, if $v(\psi) = 1$ for every $\psi \in \Gamma$, then $v(\varphi) = 1$.

Define $\varphi * \psi := \neg\varphi \to \psi$.

(a) Show that, for any fuzzy valuation $v : \mathsf{Prop} \to [0, 1]$, we have

$$v(\varphi * \psi) = \min(1, v(\varphi) + v(\psi)).$$

(b) Define $p^1 := p$ and $p^{n+1} := p^n * p$. And

$$\Gamma := \{\neg p \to q\} \cup \{p^n \to q : n \geq 1\}.$$

Show that $\Gamma \vDash_1 q$.

Hint: If $v(p) = 0$, the first conditional gives $v(q)$, and if $v(p) > 0$, find a large enough n such that $v(p^n) = 1$.

(c) Show that, for any finite $\Gamma_0 \subseteq \Gamma$, we have $\Gamma_0 \nvDash_1 q$.

(d) Conclude that $Ł_{\aleph_1}$ cannot have a sound and complete proof system.

Exercise 3 [25 points]

This exercise concerns system **C** and cumulative models in non-monotonic logic. Fix a natural number $k \geq 1$ and consider the following rule schema R_k:

$$\frac{\alpha_0 \mathrel{|\!\sim} \alpha_1,\ \alpha_1 \mathrel{|\!\sim} \alpha_2,\ \ldots,\ \alpha_{k-1} \mathrel{|\!\sim} \alpha_k,\ \alpha_k \mathrel{|\!\sim} \alpha_0}{\alpha_0 \mathrel{|\!\sim} \alpha_k} \qquad (R_k)$$

(a) Show that for any $k \geq 2$, R_k is *not* valid in the class of cumulative models.

(b) Let $\mathbf{C} + R_k$ be the proof system obtained by adding R_k to the rules of system **C**. Show that in $\mathbf{C} + R_k$ the following is a *derived* rule: for any $i, j \in \{0, \ldots, k\}$,

$$\frac{\alpha_0 \mathrel{|\!\sim} \alpha_1,\ \alpha_1 \mathrel{|\!\sim} \alpha_2,\ \ldots,\ \alpha_{k-1} \mathrel{|\!\sim} \alpha_k,\ \alpha_k \mathrel{|\!\sim} \alpha_0}{\alpha_i \mathrel{|\!\sim} \alpha_j} \qquad (R_{ij})$$

Hint: the derived rule *Equivalence* (see Definitions) might be helpful. You can assume it.

(c) A *cumulative ordered model* is a cumulative model $\mathcal{M}_C = \langle S, \ell, \prec, V\rangle$ in which the preference relation $\prec$ is a strict partial order (i.e. irreflexive and transitive). Show that R_k is valid in all cumulative ordered models.

Exercise 4 [25 points]

Explain why Tarski's account of truth for formal languages leads to a hierarchy of truth predicates. Explain why this might seem problematic or counterintuitive and then sketch a possible reply. In your answer, give at least one clear criticism of the hierarchy and one plausible response to that criticism.